Research Notes

Economic Joint Ordering of Consumable Items for a University Library

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In this paper the problem of joint ordering of consumable items for a university library is formulated. A simple heuristic method is proposed for determining the most economical ordering policy. An example involving two items is solved to illustrate the method.

A modern library consumes a large number of items in its day-to-day operations, i.e., loan slips, interlibrary loan forms, photocopying paper of different kinds and sizes, standard stationery items, order forms, etc. Very often a number of items are procured from a single supplier. For example, our local public library purchases photocopying paper in two sizes from one supplier. The demand for the photocopying paper in the two sizes is fairly constant over time. The problem facing the library management is how to procure photocopying paper in the most economical manner.

Items that are purchased exclusively from a single supplier can be ordered in two simple ways: (1) each item is ordered on its own, and (2) items are ordered jointly.

Whenever a purchase order is initiated, a fixed cost of placing an order is incurred. This fixed cost primarily consists of administrative costs of preparing a purchase order. Some additional cost may be incurred depending on the particular item(s).

Therefore, in the first instance, where each item is ordered on its own, every time an item is ordered the fixed cost of placing the purchase order and the variable cost of ordering that particular item must be incurred. On the other hand, in the second instance, where items are ordered jointly, the cost of placing the order consists of the fixed cost of placing the order plus the total of individual ordering cost of the items included in the purchase order. Therefore, as a result of ordering items jointly, considerable saving in the total ordering cost can be achieved. In the operations research/management science literature, a considerable number of publications exist that deal with the problem of determining an economical ordering policy (see Shu,[1] Goyal,[2,3] Silver,[4] and Kaspi and Rosenblatt[5].)

For the consumable items having a fairly constant rate of demand the Economic Order Quantity Model (EOQ model) is considered to be most appropriate for application in the context of a library inventory of consumable items.

Let

S = fixed cost associated with a replenishment;

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n = number of consumable items ordered from the supplier and for the i^{th} item;
S_i = variable cost of including the item on the replenishment order;
D_i = demand per year;
h_i = stock holding cost per item per year.

In the first instance, where each item is ordered on its own, the EOQ and the minimum annual cost (MAC(EOQ)), for the i^{th} items are given by formulas 1 and 2. The minimum annual cost for all the consumable items is given by formula 3.

In the second instance where items are ordered jointly, let T represent the time interval between replenishments. The order quantity for the i^{th} item will be TD_i. Hence the annual cost, Z, of ordering and stock holding in case of joint replenishment is given by formula 4.

The total annual cost has a least value at $T = T_o$ as given in formulas 5 and 6.

$$(EOQ)_i = \sqrt{\frac{2D_i (S+S_i)}{h_i}} \qquad (1)$$

$$(MAC(EOQ))_i = \sqrt{2D_i h_i (S+S_i)} \qquad (2)$$

$$(MAC(EOQ)) = \sum_{i=1}^{n} \sqrt{2D_i h_i (S+S_i)} \qquad (3)$$

$$Z = \frac{(S + \sum_{i=1}^{n} S_i)}{T} + \frac{T}{2} \sum_{i=1}^{n} D_i h_i \qquad (4)$$

$$T_o = \sqrt{\frac{2(S + \sum_{i=1}^{n} S_i)}{\sum_{i=1}^{n} D_i h_i}} \qquad (5)$$

$$(MAC(JOINT)) = \sqrt{2(S + \sum_{i=1}^{n} S_i) \sum_{i=1}^{n} D_i h_i} \qquad (6)$$

$$(EOQ(JOINT))_i = D_i \sqrt{\frac{2(S + \sum_{i=1} S_i)}{\sum_{i=1}^{n} D_i h_i}} \qquad (7)$$

The economic order quantity for the i^{th} item is given by formula 7.

If the total annual cost (MAC(JOINT)) as obtained from (6) is lower than the (MAC(EOQ)) evaluated from (3), then it is more economical to order items jointly.

Very often, the $(EOQ(JOINT))_i$ for some items may be lower than the EOQ when the item is ordered on its own without incurring the fixed cost, S, (in such a situation

$$\left[(EOQ)_i = \sqrt{2D_iS_i/h_i} \right].$$

For these items it makes sense to order less frequently, every second replenishment or every third, and so on. In short, the i^{th} item may be ordered in every K_i^{th} replenishment where $K_i = 1, 2, 3$, and so on. In order to determine the value of K_i, we select the nearest non-zero integer number from the ratio shown as formula 8. See Silver[6] for determining the value of K_i.

The time interval between replenishments, the order quantity for the i^{th} item and the (MAC(JOINT)) are given by formulas 9, 10, and 11.

Therefore, for items purchased from a single supplier, the following steps should be implemented for determining the economic ordering policy.

A SYSTEMATIC PROCEDURE FOR DETERMINING THE ECONOMIC POLICY

Step 1. For each item $i = 1, 2, \ldots, n$, evaluate $(EOQ)_i$ from (1), and (MAC(EOQ)) from (3).

Step 2. Obtain $(EOQ(JOINT))_i$ from (7), evaluate

$$\frac{\sqrt{2D_iS_i/h_i}}{(EOQ(JOINT))_i}$$

and select the nearest non-zero integer as the value of K_i for $i = 1, 2, \ldots, n$.

Step 3. Determine (MAC(JOINT)) from (11), if (MAC(EOQ)) $\leq$ (MAC(JOINT)) then select $(EOQ)_i$ as the order quantity for each item and the policy is to order items individually. If (MAC(EOQ)) $>$ (MAC(JOINT)) then items are ordered jointly.

AN EXAMPLE

A library orders photocopying paper in two sizes from a supplier. The various es-

$$K_i = \frac{\sqrt{2D_iS_i/h_i}}{(EOQ(JOINT))_i} \quad (8)$$

$$T(JOINT) = \sqrt{\frac{2(S + \sum_{i=1}^{n} S_i/K_i)}{\sum_{i=1}^{n} D_iK_ih_i}} \quad (9)$$

$$(EOQ(JOINT))_i = D_i \bullet T(JOINT) \quad (10)$$

$$(MAC(JOINT)) = \sqrt{2(S + \sum_{i=1}^{n} S_i/K_i) \sum_{i=1}^{n} D_iK_ih_i} \quad (11)$$

Step 1. From (1):

$$(EOQ)_1 = \sqrt{\frac{2D_1(S + S_1)}{h_1}} = \sqrt{\frac{2 \times 1000(20 + 8)}{1}}$$

$$= 237 \text{ boxes}$$

$$(EOQ)_2 = \sqrt{\frac{2D_2(S + S_2)}{h_2}} = \sqrt{\frac{2 \times 150(20 + 8)}{1.1}}$$

$$= 87 \text{ boxes}$$

From (3):

$$(MAC(EOQ)) = \sqrt{2_1D_1h_1(S + S_1)} + \sqrt{2D_2h_2(S + S_2)}$$

$$= \sqrt{2 \times 1000 \times 1(20 + 8)} + \sqrt{2 \times 150 \times 1.1(20 + 8)}$$

$$= 236.64 + 96.12$$

$$= \$332.76 \text{ per year}$$

Step 2. From (7):

$$(EOQ(JOINT))_1 = D_1\sqrt{\frac{2(S + S_1 + S_2)}{D_1h_1 + D_2h_2}} = 1000\sqrt{\frac{2(20 + 8 + 8)}{(1000\times1 + 150\times1.1)}}$$

$$= 249 \text{ boxes.}$$

$$(EOQ(JOINT))_2 = D_2\sqrt{\frac{2(S + S_1 + S_2)}{D_1h_1 + D_2h_2}}$$

$$= 37 \text{ boxes.}$$

$$\frac{\sqrt{2D_1S_1/h_1}}{(EOQ(JOINT))_1} = \frac{\sqrt{2 \times 1000\ 8/1}}{249} = 0.51, \text{ hence } K_1=1$$

$$\frac{\sqrt{2D_2S_2/h_2}}{(EOQ(JOINT))_2} = \frac{\sqrt{2\times150\times8/1.1}}{37} = 1.26, \text{ hence } K_2=1$$

Step 3. From (11):

$$(MAC(JOINT)) = \sqrt{2(S+\frac{S_1}{K_1}+\frac{S_2}{K_2})(D_1h_1K_1+D_2h_2K_2)}$$

$$= \sqrt{2(20+8+8)(1000\times1\times1+150\times1.1\times1)}$$

$$= \$289.62 \text{ per year}$$

FIGURE 1

$$T(JOINT) = \sqrt{\frac{2(S + \frac{S_1}{K_1} + \frac{S_2}{K_2})}{D_1h_1K_1 + D_2h_2K_2}} = \sqrt{\frac{2(20 + 8 + 8)}{1000 + 165}}$$

$$= 0.249 \text{ years}$$

$$(EOQ(JOINT))_1 = 249 \text{ boxes; } (EOQ(JOINT))_2 = 37 \text{ boxes}$$

FIGURE 2

timates for the problem are given below:

$S = \$20$ per order

For size 1 : $i = 1$
$D_1 = 1000$ boxes/per year
$S_1 = \$8$ per order
$h_1 = \$1$ per box per year

For size 2 : $i = 2$
$D_2 = 150$ boxes per year
$S_2 = \$8$ per order
$h_2 = \$1.10$ per box per year

We apply the three steps to this problem as shown in figure 1.

Because (MAC(JOINT)) is lower than (MAC(EOQ)), the economic policy is to order jointly. The orders are placed at intervals obtained from (9). (See figure 2.) The reduction in cost as a result of ordering jointly

$$= (MAC(EOQ)) - (MAC(JOINT))$$
$$= 332.76 - 289.62$$
$$= \$43.14 \text{ per year}$$

The percentage reduction in cost

$$= \frac{100 \times [(MAC(EOQ)) - (MAC(JOINT))]}{(MAC(EOQ))}$$

$$= \frac{100(332.76 - 289.62)}{332.76}$$

$$= 12.96\%$$

CONCLUDING REMARKS

As a result of ordering items from a single supplier in an economical manner, significant cost savings can be achieved. The procedure given in this paper can help in achieving such savings.

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